Advanced and Contemporary Topics in Macroeconomics I

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Class Lecture Note 6

Consumption/Saving

Based on David Romer (2012)

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Lecture 6
Consumption/Saving

A. Introduciton: Consumption/Saving

Aim: understand the micro foundation for grwoth.

- Consumption and Investment decisions are crucial for growth and fluctuation
- With regard to grwoth the division of resources between consumption and various investment (physical capitlal, human capital and R&D) is crucial for growth
- Thus it is impetrative to learn decisions about such issues (such micro foundations issues)
  - It is also imperative to note that a decision about consumption is closely related to saving which, in turn, is central to the grwoth process as we have seen in SS and RCK as well as the Endogenous growth models.

Introduction ..... Cont'd

- In addition there are two other major reason for studying consumption and investment:
  - First, they introduce important issues involving financial markets (which in turn affect consumption and investment)
  - Second, insightful empirical work on macroeconomy in the last two to three decades is related to consumption and investment
- Moreover in most countries (in DCs) consumption is about 75% of GDP, and more in LDCs – thus the need to understand this big aggregate.

B. Consumption under Certainity:Life-Cycle/Permanent-Income Hypothesis

- **Assumptions**
- Assume an individual who lives T years and her life time utility is given by

$$U = \sum_{t=1}^{T} u(C_t) \qquad u'(\bullet) > 0 \quad u''(\bullet) < 0 \qquad (1)$$

- The individual has initial wealth (A_0), labour income Y_1 to Y_T. She can borrow at an exogenous interest rate (assumed zero; individual's discount rate is also assumed zero) with no Ponzi scheme. The budget constraint is thus given by

$$\sum_{t=1}^{T} C_t \leq A_0 + \sum_{t=1}^{T} Y_t \qquad (2)$$

B. Consumption under Certainity:Life-Cycle/Permanent-Income Hypothesis

- **Behavior**
- Since the marginal utility is always positive, the individual satisfies the budget constraint with equality and the Lagrangian for maximization is given by:

$$L = \sum_{t=1}^{T} u(C_t) + \lambda\left(A_0 + \sum_{t=1}^{T} Y_t - \sum_{t=1}^{T} C_t \right) \quad (3)$$

- The FOC for C_t is

$$u'(C_t) - \lambda = 0 \implies u'(C_t) = \lambda \quad (4)$$

- Since [4] holds every period the MU of consumption is constant. Since consumption uniquely determines the MU, consumption is, hence, constant. Thus $C_1 = C_2 .. = C_T$, substituting this in the budget constraint offers Eqn [5]

Consumption under Certainity...Cont'd

$$C_t = \frac{1}{T}\left(A_0 + \sum_{t=1}^{T} Y_t\right) \quad \text{for all t.} \qquad (5)$$

- Eqn [5] states that the individual divides his/her life time resources equally among each period of life**
- **<u>Implications</u>**
- This analysis says that consumption in a given period is determined not by income in that period but by life time income [or permanent income]
- In the terminology of Friedman (1957) the RH of Eqn [5] is the permanent income (PI) & the difference b/n PI and current income is transitory income.
- To see this distinction let us assume a windfall rise (Z) in current income in the first period.

Consumption under Certainity ...Cont'd

- Although Z raises current income by Z, it raises permanent income only by Z/T.
  - If the hh's planning horizon [T] is large the effect on PI and hence consumption is negligible. {Ex, thus, the implication is say a temporary tax cut may not affect consumption}
- We can also infer from this analysis that although the time pattern of income is not important for consumption, it is for saving, as give by [6]

$$S_t = Y_t - C_t$$

$$S_t = \left(Y_t - \frac{1}{T}\sum_{t=1}^{T} Y_t \right) - \frac{1}{T} A_0 \qquad (6)$$

- Note here that S is higher when Y is higher relative to its average – ie when transitory income is high (and the converse, negative saving, if the reverse happens)

Consumption under Certainity ...Cont'd

- Thus, individuals use saving and borrowing to smooth consumption
- This is the key idea of the life-cycle/permanent income hypothesis of Modiglani and Brumberg (1954) and Firedman (1957).
- **What is Saving? then**
- From both theories above saving is simply future consumption/or in classical language "abstinence from current consumption".
- Thus, the decision between saving and consumption is driven by preference b/n present and future consumption.
  - Given this view do you think the poor saves less than the rich because its income is really low? What about 'Keeping up with joneses'? Does it hold in this framework?**

Consumption under Certainity ...Cont'd

Empirical Application: Understanding Estimated Consumption Functions

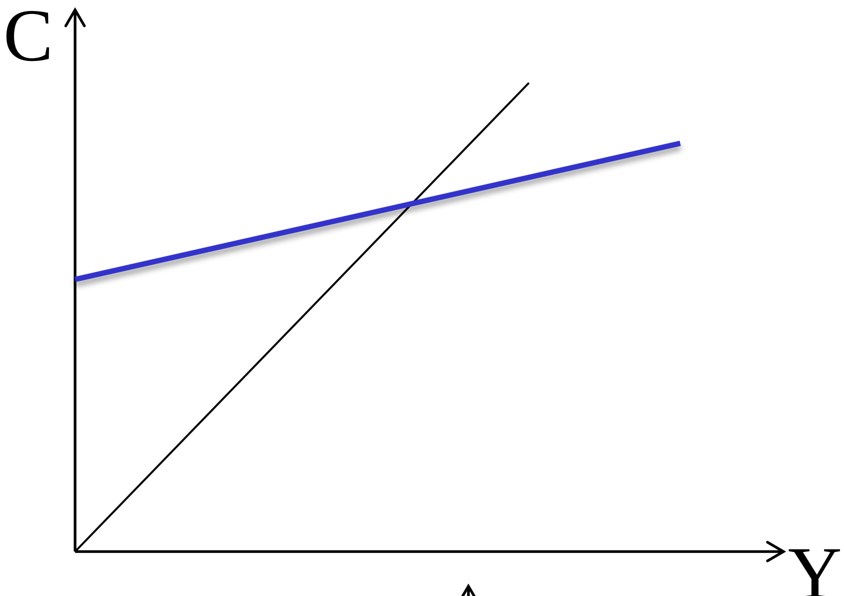
- The traditional Keynesian consumption function posits that consumption is determined by current disposable income.
  - And Keynes (1936) claimed this relationship is stable.
  - Asserted that higher income leads to higher proportion of income to be saved.
- Contrary to Keynes's claim estimated consumption function came up with a different story. These different results are given by Figure 1, panel (a) to (c)

Consumption under Certainity ...Cont’d

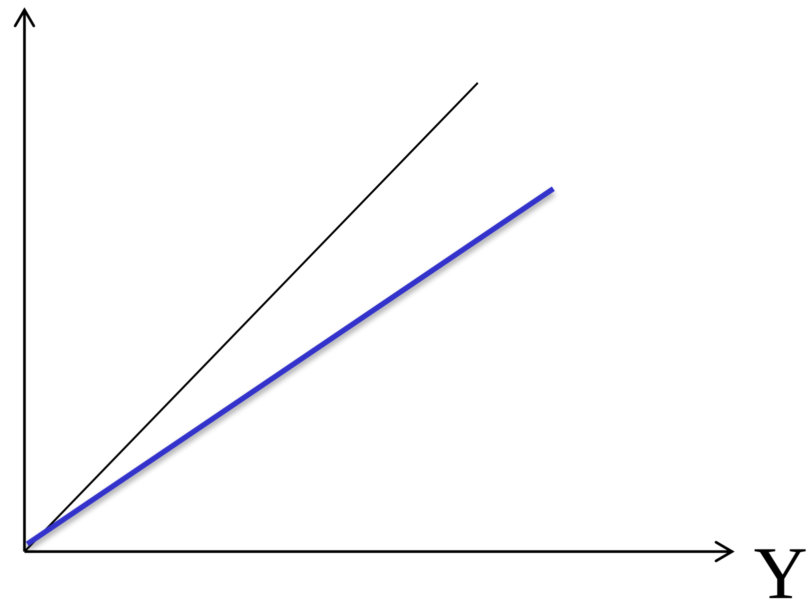
- Panel (a): Across hh at a point in time the relationship is the type that Keynes postulated.
- Panel (b): But, within a country over time, aggregate consumption is essentially proportional to aggregate income.
- Panel (c): Further the cross-section consumption function differ across groups (eg the slop of the consumption function is the same for blacks and whites but the intercept is higher for whites in USA)
- As noted by Friedman (1957) a permanent income hypothesis provides an explanation for these findings and hence can reconcile them.

Consumption under Certainity ...Cont'd

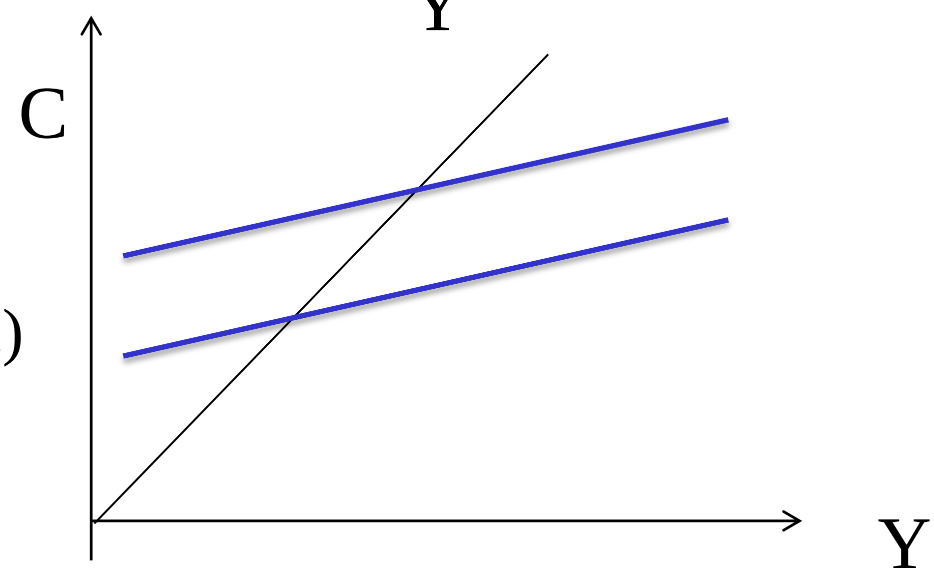
- panel (a)



Panel (b)



Panel (c)



Consumption under Certainity ...Cont'd

- Consider the regression of consumption on current income.

$$C_i = a + bY_i + e_i \qquad (7)$$

- In a univariate regression this implies

$$\hat{b} = \frac{Cov(Y, C)}{Var(Y)}$$
$$= \frac{Cov(Y^P + Y^T, Y^P)}{Var(Y^P + Y^T)}$$
$$= \frac{Var(Y^P)}{Var(Y^P) + Var(Y^T)} \qquad (8)$$

- We have used here consumption equals to PI & in the last expression PI and TI are uncorrelated.

Consumption under Certainity ...Cont'd

- The regression result implies that the constant is given by (for the mean TI is zero):

$$\hat{a} = \overline{C} - \hat{b}\overline{Y}$$
$$= \overline{Y}^{P} - \hat{b}\left(\overline{Y}^{P} + \overline{Y}^{T}\right)$$
$$= \left(1 - \hat{b}\right)\overline{Y}^{P} \qquad (9)$$

- Thus, the PI hypothesis predicts that the key determinant of the slope of an estimated consumption function, b, is relative variation of permanent and transitory income.
  - An increase in current income (CI), increase consumption only to extent it increase PI substantially (hence CI impacts)
  - If CI impact on PI is small, hence negligible impact on consumption.

Consumption under Certainity ...Cont'd

- This explains the previous graphs.
  - Across hh, much of the variation in income reflects TI (such as unemployment, hh being in different life-cycle etc) hence the estimated slope is substantially less than 1.
  - Over time, in contrast, variation in income shows long-run growth/permanent increase in the economy's resources, thus the slope coefficient is closer to 1.
  - For black and white the relative variation of PI and TI income is similar hence similar slope. But blacks average income is lower and hence lower intercept term, a. [So difference in average income of blacks and whites is the reason for different consumption pattern, not test or anything]
- Note also the finding of Kuznets (1942) where APC is constant over the long-run (against Keynes' where APC declines as Y increase [APC>MPC] Recall the graph (similar to panel c)!.

C. Consumption under Uncertainity: The Random-Walk Hypothesis

- Let us extend our analysis so far to account for uncertainly
- We will assume the instantaneous utility function, u(•), is quadratic. Thus, individuals maximizes:

$$E(U) = E\left[\sum_{t=1}^{T}\left(C_t - \frac{a}{2}C_t^2\right)\right] \qquad a > 0 \qquad (10)$$

- As before no-ponzi scheme thus the budget constraint is given by Eqn [2].
- To investigate the hh behavior we use the Euler equation approach of the RCK lecture.
  - Specifically suppose the individual choses the first period consumption optimally, given the information available (and do so in future periods too).

Consumption under Uncertainity ...Cont'd

- Now consider, a reduction in C_1 of dC from the value the individual has chosen and an equal increase in consumption at some future date from the value he/she would have chosen.
- If the individual is optimizing, a marginal change of this type doesn't affect expected utility.
  - Since the MU of consumption in period 1 is $1-aC_1$, the change has a utility cost of $(1-aC_1)dC$.
  - Sine the MU of period-t consumption is $1-aC_t$, the change has an expected utility benefit of $E_1[1-aC_t]dC$, where E(•) is expectation conditional on information available at period 1. Thus if the individual is optimizing we have Eqn[11] below

Consumption under Uncertainity ...Cont'd

$$1 - aC_1 = E_1[1 - aC_t] \quad \text{for t} = 2, 3\ldots\text{T}a \qquad [11]$$

Since $E_1[1 - aC_t]$ equals $1 - aE_1[C_t]$

$$C_1 = E_1[C_t] \qquad [12]$$

- Since the individual knows her life time consumption satisfies the budget constraint [2] with equality, the expectation of the two sides of the constraint will be,

$$\sum_{t=1}^{T} E_1[C_t] = A_0 + \sum_{t=1}^{T} E_1[Y_t] \qquad [13]$$

- [12] implies that the LHS of [13] is TC_1, substituting this in [13] and dividing by T gives,

$$\mathbf{C}_1 = \frac{\mathbf{1}}{\mathbf{T}}\left(\mathbf{A}_0 + \sum_{t=1}^{T} E_1[Y_1] \right) \qquad [14]$$

- The individual consumes 1/T of her expected life time resources

Consumption under Uncertainity ...Cont'd

Implications

- Eqn 12 showed that expectation at period 1 of C_2 equals C_1, generalizing this, expected next period consumption equals current consumption. Thus,

$$C_t = E_{t-1}[C_t] + e_t \qquad [15]$$

where expectation of e_t as of period t - 1 is 0, thus $E_{t-1}[C_t] = C_{t-1}$

$$C_t = [C_{t-1}] + e_t \qquad [16]$$

- This is the famous Hall's (1978) result that the life-cycle/ permanent income hypothesis implies that consumption follows a random walk.
- The intuition of this result is that if consumption is expected to change, the individual can do better job of smoothing consumption
  - Thus the individual adjust his or her current consumption to the point where consumption is not expected to change

Consumption under Uncertainity ...Cont'd

- Our analysis can be used to find what determined the change in consumption, or the e? Consider the change from period 1 to 2. Using the same reasoning as in Eqn [14] implies that C_2=1/(T-1) of the individual's expected remaining life time resources.

$$C_2 = \frac{1}{T-1}\left(A_1 + \sum_{t=2}^{T} E_2[Y_t]_t\right)$$

$$C_2 = \frac{1}{T-1}\left(A_0 + Y_1 - C_1 + \sum_{t=2}^{T} E_2[Y_t]_t\right) \qquad [17]$$

- Where the 2nd line uses $A_1=A_0+Y_1-C_1$.
- We can rewrite the expectation as of period 2 of income over the reminder of life, $\sum_{t=2}^{T} E_2\,[Y_t]$, = as expectation of this quantity as of period 1, $\sum_{t=2}^{T} E_1\,[Y_t]$, plus the information learned between period 1 &2. $\sum_{t=2}^{T} E_2\,[Y_t] - \sum_{t=2}^{T} E_1\,[Y_t]$,With this we can write Eqn [17] as

Consumption under Uncertainity ...Cont'd

- [ie. Using this reasoning & $A_1=A_0+Y_1-C_1$, similar to Eqn [14]] Eqn [17] becomes,

$$C_2 = \frac{1}{T-1}\left(A_0 + Y_1 - C_1 + \sum_{t=2}^{T} E_1[Y_t] + \left(\sum_{t=2}^{T} E_2[Y_t] - \sum_{t=2}^{T} E_1[Y_t]\right)\right) \qquad [18]$$

$$C_2 = \frac{1}{T-1}\left(TC_1 - C_1 + \left(\sum_{t=2}^{T} E_2[Y_t] - \sum_{t=2}^{T} E_1[Y_t]\right)\right)$$

$$C_2 = C_1 + \frac{1}{T-1}\left(\sum_{t=2}^{T} E_2[Y_t] - \sum_{t=2}^{T} E_1[Y_t]\right) \qquad [19]$$

Consumption under Uncertainity ...Cont’d

- Eqn 19 shows that the change in consumption between the two periods equals the change in individual's estimate of her life time income divided by the # periods of life remaining
- Eqn [19] shows the individual consumes the amount she would if her future income were certain to equal its means as shown in eqn 14 (ie uncertainty has no impact on consumption).
- To see the intuition for this “certainty-equivalent behaviour” consider the Euler equation relating consumption in the two periods

$$u'(C_1) = E_1[u'(C_2)] \qquad [20]$$

- When utility is quadratic, MU is linear, thus E[MU(C)] is the MU(E[C]) [ie $E_1[1-aC_2]=1-aE_1[C_2]$, thus Eqn [20] becomes**

$$u'(C_1) = u'\left(E_1[(C_2)]\right) \qquad [21]$$

- This implies $[C_1=E_1[C_2]$. Thus the analysis shows the quadratic utility is the source of the ‘certainty-equivalent behaviour”

C. Emperical Application: Some tests on The Rndom-Walk Hypotheis

- The traditional view of consumption over the business cycle implies that when output declines, consumption declines but is expected to recover
  - implies that there are predictable movements in consumption.
- Hall's extension of the permanent-income hypothesis, in contrast, predicts that when output declines unexpectedly, consumption declines only by the amount of the fall in permanent income; as a result, it is not expected to recover and fall with each change in income
- The hypothesis that consumption responds to predictable income movements (which is in line with Keynes) is referred to as *excess sensitivity* of consumption proposition (Flavin, 1981)

Emperical Application....Cont'd

A) Campbell and Mankiw's Test Using Aggregate Data

- The RW hypothesis implies that consumption is unpredictable hence no variable at time t is useful for prediction:
  - One approach to test the random-walk hypothesis is therefore to regress the change in consumption on variables that are known at $t-1$.
  - If the random-walk hypothesis is correct, the coefficients on the variables[eg income] should not differ systematically from zero.
- This is the approach that Hall took in his original work.
  - He found that neither income nor consumption can predict the change in consumption.
  - He did find, however, that lagged stock-price movements have statistically significant predictive power for the change in consumption.

Emperical Application....Cont'd

- The disadvantage of this approach is that the results are hard to interpret
  - For example, Hall's result that lagged income does not have strong predictive power for consumption could arise not because predictable changes in income do not produce predictable changes in consumption, but because lagged values of income are of little use in predicting income movements.
- Campbell and Mankiw (1989) therefore use an instrumental-variables approach to test Hall's hypothesis against a specific alternative.

$$C_t - C_{t-1} = \lambda(Y_t - Y_{t-1}) + (1-\lambda)e_t$$
$$\equiv \lambda \mathrm{Z_t} + \mathrm{v_t} \qquad [22]$$

- *Zt* and *vt* are almost surely correlated [ie Times when income increases greatly are usually also times when households receive favorable news about their total lifetime incomes]

Emperical Application....Cont'd

- Thus estimating [22] by OLS leads to estimates of λ that is biased upward. The solution is to use IV rather than OLS. $C_t - C_{t-1} = \lambda\hat{Z}_t - +\lambda(Z_t - \hat{Z}_t) + v_t$ [23]
- Using this approach they found that:
  - lagged changes in income have almost no predictive power for future changes [in income]. This suggests that Hall's failure to find predictive power of lagged income movements for consumption is not strong evidence against the traditional view of consumption.
  - As a base case, they therefore used lagged values of the change in consumption as instruments. When three lags are used, the estimate of λ is 0.42, with a standard error of 0.16; when five lags are used, the estimate is 0.52, with a standard error of 0.13. Other specifications yield similar results.

Emperical Application....Cont'd

- Thus Campbell and Mankiw's estimates suggest significant departures from the predictions of the random-walk model:
  - consumption appears to increase by about fifty cents in response to an anticipated one-dollar increase in income, and the null hypothesis of no effect of change income on consumption is strongly rejected.
- At the same time, the estimates of λ are far below 1. Thus the results also suggest that the permanent-income hypothesis is important to understanding consumption.

Emperical Application....Cont'd

B) Shea's Test using Household Data

- Shea regressed consumption growth on a measure of expected wage growth; the permanent-income hypothesis predicts that the coefficient should be 0.9
- The estimated coefficient is in fact 0.89, with a standard error of 0.46. Thus Shea also finds a quantitatively large (though only marginally statistically significant) departure from the random-walk prediction.
- One reason that consumption might not follow a random walk is that the assumption of borrowing as you wish might fail—that is, that households might face *liquidity constraints*.[if u not borrow current income could be less than permanent income; so consn is determine by current income – implies predictable change in income brings about predictable change in consumption.]

Emperical Application....Cont'd

- Many other researchers have obtained findings similar to Campbell and Mankiw's and Shea's.
- For example, Parker (1999), Souleles (1999), Shapiro and Slemrod (2003), and Johnson, Parker, and Souleles (2006) identify features of government policy that cause predictable income movements.
  - All these authors find that the predictable changes in income resulting from the policies are associated with substantial predictable changes in consumption.
  - This pattern appears to break down, however, when the predictable movements in income are large and regular
  - Etc etc Read! more such articles

D. The Interst Rate and Saving

- If we extend our consumption with certainty to a constant interest rate (r), the individuals budget constrain, discounted to period 0 will be given by

$$\sum_{t=1}^{T}\frac{1}{(1+r)^t}C_t \le A_0 + \sum_{t=1}^{T}\frac{1}{(1+r)^t}Y_t \qquad [24]$$

- Similarly, as before, the constant-relative-risk-aversion utility with a personal discount rate ρ function could be given by [25]

$$U = \sum_{t=1}^{T}\frac{1}{(1+\rho)^t}\frac{C_t^{1-\theta}}{1-\theta} \qquad [25]$$

D. The Interst Rate and Saving

- Doing our usual experiment of decreasing consumption at t and increasing it at t+1, with interest rate (r+1) and the MU for t &t+1gives [26], which could be simplified as [27]

$$\frac{1}{(1+\rho)^{t}} C_t^{-\theta} = (1+r) \frac{1}{(1+\rho)^{t+1}} C_{t+1}^{-\theta} \qquad [26]$$

$$\frac{C_{t+1}}{C_t} = \left(\frac{1+r}{1+\rho} \right)^{1/\theta} \qquad [27]$$

- This analysis implies that once we allow for the possibility that the real interest rate and the discount rate are not equal, consumption need not be a random walk:
  - consumption is rising over time if *r* exceeds ρ and falling if *r* is less than ρ..

The Interst Rate and Saving.....Cont'd

- Hansen and Singleton (1983), Hall (1988b), Campbell and Mankiw (1989), and others therefore examine how much consumption growth responds to variations in the real interest rate.
  - For the most part they find that it responds relatively little, which suggests that the intertemporal elasticity of substitution is low (that is, θ is high).
  - I think similar results are found in Ethiopian and African countries (check some MSc thesis)
  - ----"-------

E. Consumption and Risky Assets :The Consumption Captial-Asset Price Model [CAPM]

- Individuals can invest in many assets which have uncertain returns. If we extend our model to multiple assets and risk, new issues about hh behavior and asset markets will emerge – this is discussed here

The Condition for Individual Optimization

- Consider an individual reducing consumption in period *t* by an infinitesimal amount and using the resulting saving to buy an asset, *i*, that produces a potentially uncertain stream of payoffs, *Di t*+1, *Di t*+2, ...
- If the hh is optimizing, the marginal utility she forgo from the reduced consumption in period *t* must equal the expected sum of the discounted marginal utilities of the future consumption provided by the asset's payoffs.

$$u'(C_t) = \frac{1}{1+\rho} E\left[(1+r_{t+1}^i)u'(C_{t+1})\right] \qquad \text{for all i} \qquad [28]$$

Consumption and Risky Assets...Cont'd

- Since the expectation of the product of two variables equals the product of their expectations plus their covariance, we can rewrite this expression as

$$u'(C_t) = \frac{1}{1+\rho}\left\{\left[E_t[1+r_{t+1}^i]E_t[u'(C_{t+1})] + Cov_t(1+r_{t+1}^i, u'(C_{t+1})\right]\right\} \text{ for all } \quad i \qquad [29]$$

- If we assume that utility is quadratic, $u(C) = C - aC^2/2$, then the marginal utility of consumption is $1 - aC$. Using this to substitute for the covariance term in [29], we obtain

$$u'(C_t) = \frac{1}{1+\rho}\left\{\left[E_t[1+r_{t+1}^i]E_t[u'(C_{t+1})] - aCov_t(1+r_{t+1}^i, C_{t+1})\right]\right\} \qquad [30]$$

Consumption and Risky Assets...Cont'd

- Eqn [30] implies that in deciding whether to hold more of an asset, the individual is not concerned with how risky the asset is: the variance of the asset's return does not appear in Eqn [30].
- Eqn [30] implies that the aspect of riskiness that matters to the decision of whether to hold more of an asset is the relation between the asset's payoff and consumption.
  - Suppose the hh can buy a [risky] asset whose expected return is equal to that of risk-free asset that she already could, then…
  - In buying this new [risky] assets, if the payoff to the new asset is typically high when the marginal utility of consumption is high (that is, when consumption is low), buying one unit of the asset raises expected utility by more than buying one unit of the risk-free asset.

Consumption and Risky Assets...Cont'd

- Thus, the hh can raise her expected utility by buying the new asset
- As the hh invests more in the new asset his consumption comes to depend on the assets' payoff and the covariance b/n consumption and this payoff becomes less negative
- In the example we are considering, since the asset's expected return equals the risk-free rate, the individual will invests in the asset until the covariance of its return with consumption reaches zero

• This discussion implies that hedging risks is crucial to optimal portfolio choices

- A steelworker whose future labor income depends on the health of the U.S. steel industry should avoid assets whose returns are positively correlated with U.S. steel companies. Instead the worker should invest in assets whose returns move inversely with the health of the U.S. steel industry, such as foreign companies.

Consumption and Risky Assets...Cont'd

- One implication of this analysis is that individuals should exhibit no particular tendency to hold shares of companies that operate in the individuals' own countries.
  - In reality though, individuals' portfolios are very heavily skewed toward domestic companies (French and Poterba, 1991). This pattern is known as *home bias*.
- Thus, this discussion about consumption and equity leads to the *consumption capital-asset pricing model,* or *consumption CAPM which may shed light on this issue*.

Consumption and Risky Assets...Cont'd

The Consumption CAPM

- The discussion thus far takes assets' expected returns as given. But individuals' demand for assets determine these expected returns. Solving Eqn [30] for expected return,

$$E_t(1+r_{t+1}^i) = \frac{1}{E_t[u'(C_{t+1})]}\left[(1+\rho)u'(C_t) + aCov_t(1+r_{t+1}^i, C_{t+1})\right] \qquad [31]$$

- Equation [31] states that the higher the covariance of an asset's payoff with consumption, the higher its expected return must be.
- We can simplify [31] by considering the return on a risk-free asset. If the payoff to an asset is certain, then the covariance of its payoff with consumption is zero. Thus the risk-free rate, $\bar{r}_{t+1}$ satisfies

Consumption and Risky Assets...Cont'd

$$1+\bar{r}_{t+1} = \frac{(1+\rho)u'(C_t)}{E_t[u'(C_{t+1})]} \quad [32]$$

$$E_t[r^i_{t+1}] - \bar{r}_{t+1} = \frac{aCov_t(1+r^i_{t+1}, C_{t+1})}{E_t[u'(C_{t+1})]} \quad [33]$$

- Eqn [33] states that the expected-return premium that an asset must offer relative to the risk-free rate is proportional to the covariance of its return with consumption.
- This model of the determination of expected asset returns is known as the *consumption capital-asset pricing model*, or *consumption CAPM*
- The coefficient from a regression of an asset's return on consumption growth [or their covariance] is known as its *consumption beta*

Consumption and Risky Assets...Cont'd

- Thus the central prediction of the consumption CAPM is that the premiums that assets offer are proportional to their consumption betas (Breeden, 1979; see also Merton, 1973, and Rubinstein, 1976)

Empirical Application: The Equity-Premium Puzzle

- One of the most important implications of this analysis of assets' expected returns concerns the case where the risky asset is a broad portfolio of stocks. Using Eqn [28], and assuming CRRA utility, the Euler Eqn becomes** Eqn [34]; which could be simplified to Eqn [35]

$$C_t^{-\theta} = \frac{1}{(1+\rho)} E_t\left[(1+r_{t+1}^i)C_{t+1}^{-\theta}\right] \qquad [34]$$

$$(1+\rho) = E_t\left[(1+r_{t+1}^i)\frac{C_{t+1}^{-\theta}}{C_t^{-\theta}}\right] \qquad [35]$$

Consumption and Risky Assets...Cont'd

- For convenience let $g^c{}_{t+1}$ denote the growth rate of consumption from t to $t+1$, $(C_{t+1}/C_t)-1$, and omit the time subscripts. With this we will have

$$E_t\left[(1+r^i)(1+g^c)^{-\theta}\right]=(1+\rho) \qquad [36]$$

- To see the implications of Eqn [36], we take a second-order Taylor approximation of the left-hand side around $r = g = 0$. Computing the relevant derivatives yields:

$$(1+r)(1+g)^{-\theta} \cong 1+r-\theta g-\theta gr+\tfrac{1}{2}\theta(\theta+1)g^2 \qquad [37]$$

Consumption and Risky Assets...Cont'd

- We can rewrite [36] as,

$$\mathrm{E}[\mathrm{r}^{\mathrm{i}}] - \theta E[g^{c}] - \theta\{\mathrm{E}[\mathrm{r}^{\mathrm{i}}]E[g^{c}] + Cov(\mathrm{r}^{\mathrm{i}} g^{c})$$

$$+ \tfrac{1}{2}\theta(\theta+1)\left\{\mathrm{E}\left[g^{c}\right]^{2} + \mathrm{Var}(g^{c})\right\} \cong \rho \qquad [38]$$

- When the time period involved is short the $E[r^{i}]E[g^{c}]$ and $\left(E[g^{c}]\right)^{2}$ terms are small relative to the others, omitting this terms and solving the result for $E[r^{i}]$ yields

$$\mathrm{E}[\mathrm{r}^{\mathrm{i}}] \cong \rho + \theta E[g^{c}] + \theta Cov(\mathrm{r}^{\mathrm{i}} g^{c}) - \tfrac{1}{2}\theta(\theta+1)\mathrm{Var}(g^{c}) \qquad [39]$$

- For risk free asset [39] simplifies to [40]; also subtracting 40 form [39] yields [41]

$$\mathrm{r} \cong \rho + \theta E[g^{c}] - \tfrac{1}{2}\theta(\theta+1)\mathrm{Var}(g^{c}) \qquad [40]$$

$$\mathrm{E}[\mathrm{r}^{\mathrm{i}}] - \bar{\mathrm{r}} \cong \theta Cov(\mathrm{r}^{\mathrm{i}} g^{c}) \qquad [41]$$

Consumption and Risky Assets...Cont'd

- In a famous paper, Mehra and Prescott (1985) show that it is difficult to reconcile observed returns on stocks and bonds with Eqn [41].

- Mankiw and Zeldes (1991) report a simple calculation that shows the essence of the problem.

- [see next slide about Mankiew and Zeldes and reead these two classic articles which are in your consumption folder]

Consumption and Risky Assets...Cont'd

According to Mankiw and Zeldes (1991) study:

For the United States during the period 1890–1979 (which is the sample that Mehra and Prescott consider), the difference between the average return on the stock market and the return on short-term government debt—the equity premium—is about 6 percentage points.

➔*Over the same period, the standard deviation of the growth of consumption (as measured by real purchases of nondurables and services) is 3.6 percentage points, and the standard deviation of the excess return on the market is 16.7 percentage points; the correlation between these two quantities is 0.40.*

➔*These figures imply that the covariance of consumption growth and the excess return on the market is 0.40(0.036)(0.167), or 0.0024.*

➔*Eqn [41] therefore implies that the coefficient of relative risk aversion needed to account for the equity premium is the solution to* $0.06 = \theta(0.0024)$, *or* $\theta = 25$..

> ➔*This is an extraordinary level of risk aversion; it implies, for example, that individuals would rather accept a 17 percent reduction in consumption with certainty than risk of a 50-50 chance of a 20 percent reduction.*

Consumption and Risky Assets...Cont'd

- As Mehra and Prescott describe, & other evidences suggests that risk aversion is much lower than this.
- Such a high degree of aversion to variations in consumption makes it puzzling that the average risk-free rate is close to zero despite the fact that consumption is growing over time.
- Furthermore, the equity-premium puzzle has become more severe in the period since Mehra and Prescott identified it.
  - From 1979 to 2008, the average equity premium is 7 percentage points, which is slightly higher than in Mehra and Prescott's sample period. More importantly, consumption growth has become more stable and less correlated with returns (Romer, 2012).
- The large equity premium, when coupled with the low risk free rate, is thus difficult to reconcile with household optimization behavior we are working with. This *equity-premium puzzle* has stimulated a large amount of research, and many explanations for it with no clear resolution of the puzzle has been provided, however, so far.

F. Beyond the Permanent Income Hyphotesis/Alternative Views of Consumption

(A). **Background and the Buffer-Stock Saving**

- The permanent-income hypothesis provides appealing explanations for many important features of consumption.
  - For example, it explains why temporary tax cuts appear to have much smaller effects than permanent ones,
  - and it accounts for many features of the relationship between current income and consumption,
- Yet there are also important features of consumption that appear inconsistent with permanent-income hypothesis.
  - Eg: both macroeconomic & microeconomic evidence suggest consumption often responds to predictable changes in income.
  - And as we just saw, simple models of consumer optimization cannot account for the equity premium.

Beyond the Permanent Income .... Cont'd

- Permanent-income hypothesis also fails to explain some central features of consumption behavior:
  - Eg According to PI hypothesis consumption growth is determined by the real interest rate and the discount rate, not by the time pattern of income [no reln b/n E(C&Y growth)].
  - Carroll and Summers (1991) present extensive evidence that this prediction of the permanent-income hypothesis is incorrect
- More generally, most households have little wealth (see, Wolff, 1998). Their consumption approximately tracks their income and hence their income has large role in the determination of their consumption etc
- However, hhs have a small amount of saving that they use in the event of sharp falls in income/emergency needs: Deaton (1991) calls this *buffer-stock* saving pto

Beyond the Permanent Income .... Cont'd

- In the terminology of Deaton (1991), most households exhibit *buffer-stock* saving behavior. As a result, a small fraction of households hold the vast majority of wealth
- These failings of the permanent-income hypothesis have motivated a large amount of work on extensions or alternatives to the theory.
- Three ideas that have received particular attention are.
  - **(A) Precautionary Saving**
  - **(B) Liquidity Constraint**
  - **(C) Departures from Complete Optimization**
- We will briefly touches on some of the issues raised by these ideas to finalize our lecture.[Hopefully you will do an in-depth reading and ask are they relevant to Africa?]

Beyond the Permanent Income ...Cont'd

- **Precautionary Saving**

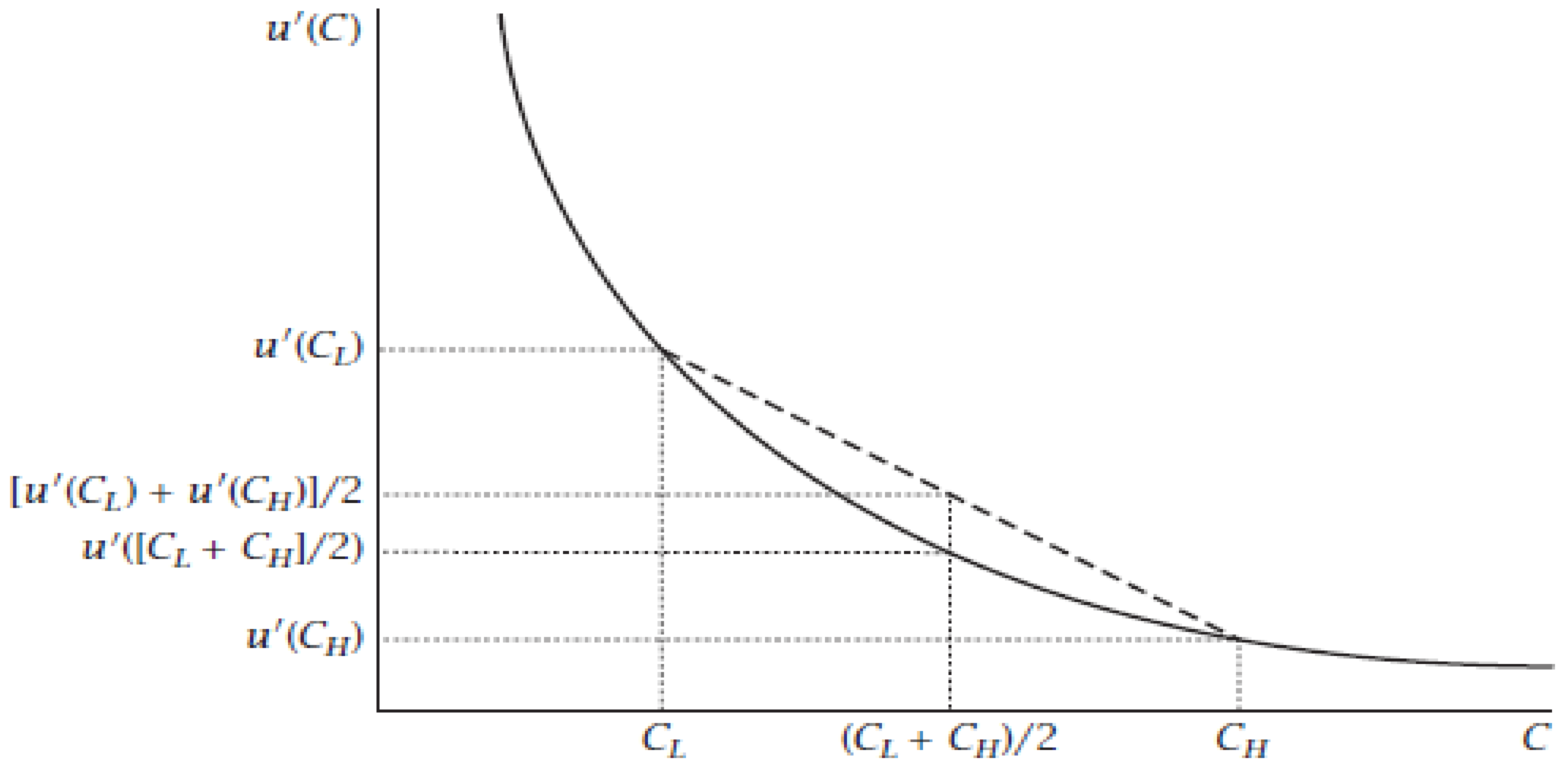
- Our derivation of the random-walk result in Section 2 was based on the assumption that utility is quadratic. Quadratic utility implies:
  - marginal utility reaches zero at some finite level of consumption and then becomes negative.
  - the utility cost of a given variance of consumption is independent of the level of consumption
- This means that,
  - since the marginal utility of consumption is declining, individuals have increasing absolute risk aversion: the amount of consumption they are willing to give up [saving!] to avoid a given amount of uncertainty about the level of consumption rises as they become wealthier.

Beyond the Permanent Income ...Cont'd

- Considering the Euler Eqn given in Eqn[2], if utility is quadratic, MU is linear and so $E_t[u'(C_{t+1})=u'E_t[C_{t+1}]$
  - In this case the Euler Eqn reduces to $C_t=E_t[C_{t+1}]$
  - But if $u'''(\bullet)$ is positive, then $u'(C)$ is a convex function of C.
  - In this case, $E_t[u'(C_{t+1})]$ exceeds $u'(E_t[C_{t+1}])$.
  - But this means that if C_t and $E_t[C_{t+1}]$ are equal, $E_t[u'(C_{t+1})]$ is greater than $u'(C_t)$, and so a marginal reduction in C_t increases expected utility. [see figure in the next slide]
- Thus the combination of a positive third derivative of the utility function and uncertainty about future income reduces current consumption, and thus raises saving. This saving is known as *precautionary saving* (Leland, 1968).

Beyond the Permanent Income ...Cont'd

Since $u''(C)$ is negative, $u'(C)$ is decreasing in C. And since $u'''(C)$ is positive, $u'(C)$ declines less rapidly as C rises. If consumption takes on only two possible values, C_L and C_H, each with probability 1/2 , the expected marginal utility of consumption is the average of marginal utility at these two values. In terms of the diagram, this is shown by the midpoint of the line connecting $u'(C_L)$ and $u'(C_H)$.



(a)

Beyond the Permanent Income ...Cont'd

- **Liquidity Constraint**
- The PI hypothesis assumes that individuals can borrow at the same interest rate at which they can save as long as they eventually repay their loans.
  - Yet the interest rate that households pay on borrowing are often much higher than the saving rate
  - Some are even unable to borrow more at any interest rate.
- Liquidity constraints can raise saving in two ways.
  - First, whenever a liquidity constraint is binding, it causes the individual to consume less than he/she otherwise would.
  - Second, even if the constraints are not currently binding, the fact that they may bind in the future reduces current consumption [Zeldes (1989)]

Beyond the Permanent Income ...Cont'd

- Example:.
  - Suppose, there is some chance of low income in the next period.
    - If there are no liquidity constraints, and if income in fact turns out to be low, the individual can borrow to avoid a sharp fall in consumption.
    - If there are liquidity constraints, however, the fall in income causes a large fall in consumption unless the individual has savings. Thus the presence of liquidity constraints causes individuals to save as insurance against the effects of future falls in income
- These points can be seen in a three-period model.

Beyond the Permanent Income ...Cont'd

- To distinguish the effects of liquidity constraints from precautionary saving,
  - assume that the instantaneous utility function is quadratic.
  - In addition, continue to assume that the real interest rate and the discount rate are zero
- Begin by considering the individual's behavior in period 2. Let A_t denote assets at the end of period t.
- Since the individual lives for only three periods, C_3 equals $A_2 + Y_3$, which in turn equals $A_1 + Y_2 + Y_3 - C_2$.
- The individual's expected utility over the last two periods of life as a function of his or her choice of C_2 is therefore:

Beyond the Permanent Income ...Cont'd

- **

$$U = (C_2 - \tfrac{1}{2} a C_2^2) + E_2[(A_1 + Y_2 + Y_3 - C_2) - a\tfrac{1}{2}(A_1 + Y_2 + Y_3 - C_2)^2] \qquad [43]$$

- The derivative of this expression wrt to C2 is:

$$\frac{\partial U}{\partial C_2} = 1 - aC_2 - (1 - aE_2[(A_1 + Y_2 + Y_3 - C_2])$$

$$= a(A_1 + Y_2 + E_2[Y_3] - 2C_2) \qquad [44]$$

- This expression is positive for $C_2 < (A_1 + Y_2 + E_2[Y_3])/2$, and negative otherwise.

Beyond the Permanent Income ...Cont'd

- Thus, as we know from our earlier analysis, if the liquidity constraint does not bind, the individual chooses $C_2 = (A_1 + Y_2 + E_2[Y_3])/2$. But if it does bind, he or she sets consumption to the maximum attainable level, which is $A_1 + Y_2$. Thus,

$$C_2 = \min\left\{\frac{A_1 + Y_2 + E_2[Y_3]}{2}, A_1 + Y_2\right\} \qquad [45]$$

- Thus the liquidity constraint reduces current consumption if it is binding

Beyond the Permanent Income ...Cont'd

- Consider the first period. If the liquidity constraint is not binding, the individual has the option of marginally raising C_1 and paying for this by reducing C_2.
  - Thus if the individual's assets are not zero, the usual Euler equation holds. With the specific assumptions we are making, this means that C_1 equals the expectation of C_2.
- But the Euler equation holds does not mean that the liquidity constraints do not affect consumption:
  - Eqn [45] implies that if the probability that the liquidity constraint will bind in the 2nd period is strictly positive, the expectation of C_2 as of period 1 is strictly less than the expectation of $(A_1 + Y_2 + E_2[Y_3])/2$.
  - A_1 is given by $A_0 + Y_1 - C_1$, and the law of iterated projections implies that $E_1[E_2[Y_3]]$ equals $E_1[Y_3]$. Thus,

Beyond the Permanent Income ...Cont'd

$$C_1 < \frac{A_0 + Y_1 + E_1[Y_2] + E_1[Y_3] - C_1}{2} \qquad [46]$$

Adding $\frac{C_1}{2}$ to both sides of Eqn [46] and then dividing by 3/2 yields

$$C_1 < \frac{A_0 + Y_1 + E_1[Y_2] + E_1[Y_3]}{3} \qquad [47]$$

- Thus even when the liquidity constraint does not bind currently, the possibility that it will bind in the future reduces consumption.
- Finally, if the value of C_1 that satisfies $C_1 = E_1[C_2]$ (given that C_2 is determined by [45]) is greater than the individual's period-1 resources, A_0+Y_1, the 1st period liquidity constraint is binding; in this case the individual consumes $A_0 + Y_1$(not C_1 as envisaged by the Euler/optimization condition)
- Thus liquidity constraints alone, like precautionary saving alone, raise saving [ie less consumption] ------"--------.

Beyond the Permanent Income ...Cont'd

- **Departure form Complete Optimization**
- The assumption of costless optimization is a powerful modeling device, and it provides a good first approximation of agent's responses to many changes.
- However, it does not provide a perfect description of how people behave.
  - There are well-documented cases of quantitatively significant, consistent and systematic departure from the predictions of standard models of utility maximization we just learned
- This may be the case with choices between consumption and saving. Because, inter alia:
  - (a)The calculations involved are complex, (b) the time periods are long, and (c) there is a great deal of uncertainty that is difficult to quantify.

Beyond the Permanent Income ...Cont'd

- So instead of complete optimizing, individuals may follow rules of thumb in choosing their consumption
  - Such rules of thumb may be the rational response to problems noted earlier
- Relying on such rules may lead households to use saving and borrowing to smooth short-run income fluctuations;
  - thus they will typically have some savings, and consumption will follow the predictions of the PI hypothesis reasonably.
  - Thus, consumption will track income fairly closely over long horizons; thus savings will typically be small.
- One specific departure that has received considerable attention is time-inconsistent preferences (eg Laibson, 1997).
  - There is considerable evidence that individuals (and animals as well) are impatient at short horizons but patient at long horizons – thus time inconsistency

Beyond the Permanent Income ...Cont'd

- Time inconsistency alone, like the other departures from the baseline model alone, cannot account for the puzzling features of consumption, however.
  - By itself, time inconsistency makes consumers act as though they are impatient: at each point in time, individuals value current consumption greatly relative to future consumption, and so their consumption is high (Barro, 1999).
  - And time inconsistency alone provides no reason for consumption to approximately track income for a large number of households, so that their savings are close to zero.
- Other factors— liquidity constraints, the ability to save in illiquid forms (may be as a restraint), and perhaps a precautionary saving motivation – seem to be needed for models with time inconsistency to fit the facts (see Angeletos, Laibson, Repetto, Tobacman, and Weinberg, 2001).

Summary & Conclusion

Two themes emerge from this discussion.

- First, no single factor can account for the main departures from the permanent-income hypothesis.
- Second, there is considerable agreement on the broad factors that must be present:
  - a high degree of impatience (from either a high discount rate or time inconsistency with a perpetually high weight on current consumption);
  - some force preventing consumption from running far ahead of income (either liquidity constraints or rules of thumb that stress the importance of avoiding debt);
  - and a precautionary-saving motive.
  - I am sure it will also be a lot more complicated in LDCs/Africa – so a lot of work/research for your guys/!!

- **END….END….END**